

Chapter 1 End of Chapter Problem Solutions

1.1

$$n = 4 \times 10^{20} \text{ molecules/in}^3$$

$$\bar{v} = \sqrt{kgRT} = 1.32 \times 10^4 \text{ in./s}$$

$$A = \frac{\pi}{4} (10^{-3} \text{ in})^2$$

$$NA = \frac{1}{4} n \bar{v} A = 1.04 \times 10^8 \text{ m/s}$$

1.2

Flow Properties: Velocity, Pressure Gradient, Stress

Fluid Properties: Pressure, Temperature, Density, Speed of Sound, Specific Heat

1.3

mass of solid= $\rho_s v_s$

mass of fluid= $\rho_f v_f$

$$X = \frac{\rho_s v_s}{\rho_s v_s + \rho_f v_f}$$

$$\Rightarrow \frac{v_f}{v_s} = \frac{1-x}{x} \frac{\rho_s}{\rho_f}$$

$$\begin{aligned} \rho_{\text{mix}} &= \frac{\rho_s v_s + \rho_f v_f}{v_s + v_f} = \frac{\rho_s + \rho_f \left(\frac{v_f}{v_s}\right)}{1 + \frac{v_f}{v_s}} \\ &= \frac{\rho_s \rho_f}{x \rho_f + (1-x) \rho_s} \end{aligned}$$

1.4

$$\text{Given } \frac{P+B}{P_1+B} = \left(\frac{\rho}{\rho_1}\right)^7$$

$$\text{For } P_1 = 1 \text{ atm} \quad \frac{\rho}{\rho_1} = 1.01$$

$$P = 3001(1.01)^7 - 3000 = 217 \text{ atm}$$

1.5

At Constant Temperature

$$\frac{P}{\rho T} = \text{constant} \Rightarrow \frac{P}{\rho} = \text{constant}$$

For 10% increase in ρ

P must also increase by 10 %

1.6

Since density varies as $\rho = \kappa P$

$$\rho_{250,000 \text{ ft}} = \rho_{S.L.} \cdot \frac{P_{250,000 \text{ ft}}}{P_{S.L.}}$$

& $\rho = nM$ (M=Molecular wt.)

$$\therefore n_{250,000} = n_{S.L.} \left[\frac{1.5 \times 10^{-7}}{2.378 \times 10^{-3}} \right] = 4 \times 10^{20} \left[\frac{1.5 \times 10^{-7}}{2.378 \times 10^{-3}} \right] = 2.5 \times 10^{16}$$

1.7

$$\vec{e}_r = |\vec{e}_r|_x \vec{e}_x + |\vec{e}_r|_y \vec{e}_y$$

$$= \cos \theta \vec{e}_x + \sin \theta \vec{e}_y$$

$$\vec{e}_\theta = |\vec{e}_\theta|_x \vec{e}_x + |\vec{e}_\theta|_y \vec{e}_y$$

$$= -\sin \theta \vec{e}_x + \cos \theta \vec{e}_y$$

Q.E.D.

1.8

$$\frac{d\vec{e}_r}{d\theta} = -\sin\theta \vec{e}_x + \cos\theta \vec{e}_y = \vec{e}_\theta$$

$$\frac{d\vec{e}_r}{d\theta} = -\cos\theta \vec{e}_x - \sin\theta \vec{e}_y = -\vec{e}_r$$

Q.E.D.

1.9 Transformation from (x,y) to (r,θ)

$$\frac{d}{dx} = \frac{dr}{dx} \frac{d}{dr} + \frac{d\theta}{dx} \frac{d}{d\theta}$$

$$\frac{d}{dy} = \frac{dr}{dy} \frac{d}{dr} + \frac{d\theta}{dy} \frac{d}{d\theta}$$

$$r^2 = x^2 + y^2$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right)$$

$$\text{so: } \frac{dr}{dx} = \frac{x}{(x^2 + y^2)^{1/2}} = \frac{r \cos \theta}{r} = \cos \theta$$

$$\frac{d\theta}{dx} = -\frac{y}{x^2 + y^2} = -\frac{r \sin \theta}{r^2} = -\frac{\sin \theta}{r}$$

$$\frac{dr}{dy} = \sin \theta \quad \frac{d\theta}{dy} = \frac{\cos \theta}{r}$$

$$\Rightarrow \frac{d}{dx} = \cos \theta \frac{d}{dr} - \frac{\sin \theta}{r} \frac{d}{d\theta}$$

$$\frac{d}{dy} = \sin \theta \frac{d}{dr} + \frac{\cos \theta}{r} \frac{d}{d\theta}$$

1.10

$$\begin{aligned}\nabla &= \frac{d}{dx} \vec{e}_x + \frac{d}{dy} \vec{e}_y + \frac{d}{dz} \vec{e}_z \\ &= \left(\cos \theta \frac{d}{dr} - \frac{\sin \theta}{r} \frac{d}{d\theta} \right) \vec{e}_x + \left(\sin \theta \frac{d}{dr} + \frac{\cos \theta}{r} \frac{d}{d\theta} \right) \vec{e}_y + \frac{d}{dz} \vec{e}_z \\ &= (\vec{e}_x \cos \theta + \vec{e}_y \sin \theta) \frac{d}{dr} + \frac{1}{r} (-\vec{e}_x \sin \theta + \vec{e}_y \cos \theta) \frac{d}{d\theta} + \vec{e}_z \frac{d}{dz}\end{aligned}$$

Thus: $\nabla = \vec{e}_r \frac{d}{dr} + \frac{1}{r} \vec{e}_\theta \frac{d}{d\theta} + \vec{e}_z \frac{d}{dz}$

1.11

$$\nabla P = \frac{dP}{dx} \vec{e}_x + \frac{dP}{dy} \vec{e}_y$$

$$\nabla P(a,b) = \rho_\infty v_\infty^2 \left\{ \left[\frac{1}{a} \cos 1 \sin 1 + 2 \right] \vec{e}_x + \frac{1}{b} (\sin 1 \cos 1) \vec{e}_y \right\}$$

$$= \rho_\infty v_\infty^2 \left\{ \left[\frac{1}{a} \frac{\sin 2}{2} + 2 \right] \vec{e}_x + \frac{1}{b} \left(\frac{\sin 2}{2} \right) \vec{e}_y \right\}$$

1.12

$$\nabla T(x,y) = T_0 e^{-1/4} \left[\frac{1}{a} \left(\cos \frac{x}{a} \cosh \frac{y}{b} \right) \vec{e}_x + \frac{1}{b} \left(\sin \frac{x}{a} \sinh \frac{y}{b} \right) \vec{e}_y \right]$$

$$\nabla T(a,b) = T_0 e^{-1/4} \left[\frac{1}{a} (\cos 1 \cosh 1) \vec{e}_x + \frac{1}{b} (\sin 1 \sinh 1) \vec{e}_y \right]$$

$$= T_0 e^{-1/4} \left[\frac{\cos 1 (e + e^{-1})}{2a} \vec{e}_x + \frac{\sin 1 (e + e^{-1})}{2b} \vec{e}_y \right]$$

$$= \frac{T_0 e^{-5/4}}{2} \left[\frac{\cos 1}{a} (1 + e^{-2}) \vec{e}_x + \frac{\sin 1}{b} (1 - e^{-2}) \vec{e}_y \right]$$

1.13

In problem 1.12 $T(x,y)$ is dimensionally homogeneous (D.H.)

$P(x,y)$ in Prob 1.11 will be D.H. if

$$P_{\infty} \sim \frac{P}{v_{\infty}^2} \quad \text{L}_{\text{Bf}} \text{ s}^2 / \text{ft}^4$$

or using the conversion factor g_c

$$1.14 \phi = 3x^2y + 4y^2$$

A scalar field is given by the function: $\phi = 3x^2y + 4y^2$

(a) Find $\nabla\phi$ at the point (3,5)

$$\phi = 3x^2y + 4y^2$$

$$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j = (6xy)i + (3x^2 + 8y)j$$

For the value of $\nabla\phi$ at the point (3,5)

$$\nabla\phi = (6xy)i + (3x^2 + 8y)j = (6)(3)(5)i + [(3)(3)^2 + (8)(5)]j = \mathbf{90i + 67j}$$

(b) Find the component of $\nabla\phi$ that makes a -60° angle with the axis at the point (3,5)

Let the unit vector be represented by $e_s = \cos\theta_i + \sin\theta_j$

$$\nabla\phi \cdot e_s = [(6xy)i + (3x^2 + 8y)j] \cdot [\cos\theta_i + \sin\theta_j]$$

At the point (3,5) this becomes:

$$\nabla\phi \cdot e_s = [90i + 67j] \cdot [\cos(-60)i + \sin(-60)j] = 90(0.5) + (67)(-0.866) = \mathbf{-13.02}$$

1.15

For an ideal gas

$$P = \frac{\rho RT}{M}$$

$$\text{From Prob 1.3: } \rho = \frac{\rho_m(1-x)}{1 - \frac{\rho_m}{\rho_s}x}$$

$$\therefore P = \frac{\rho_m(1-x)}{1 - \frac{\rho_m}{\rho_s}x} \frac{RT}{M}$$

1.16

$$\psi = \text{Arsin}\theta\left(1 - \frac{a^2}{r^2}\right)$$

$$\text{a) } \nabla \psi = \frac{d\psi}{dr} \vec{e}_r + \frac{1}{r} \frac{d\psi}{d\theta} \vec{e}_\theta = A \sin\theta \left(1 - \frac{a^2}{r^2}\right) \vec{e}_\theta$$

$$\text{b) } |\nabla \psi| = A \left[\sin^2 \theta \left(1 + \frac{a^2}{r^2}\right)^2 + \cos^2 \theta \left(1 - \frac{a^2}{r^2}\right)^2 \right]^{1/2}$$

$$|\nabla \psi|_{\max} \text{ is given by } d|\nabla \psi| = 0 \text{ or } \frac{d}{dr} |\nabla \psi| dr + \frac{d}{d\theta} |\nabla \psi| d\theta = 0$$

$$\text{Requiring } \frac{d}{dr} |\nabla \psi| = \frac{d}{d\theta} |\nabla \psi| = 0$$

$$\text{For } \frac{d}{dr} |\nabla \psi| = 0: -\sin^2 \theta \left(1 + \frac{a^2}{r^2}\right) + \cos^2 \theta \left(1 - \frac{a^2}{r^2}\right) = 0 \quad (1)$$

$$\text{And for } \frac{d}{d\theta} |\nabla \psi| = 0: \sin\theta \cos\theta \left[\left(1 + \frac{a^2}{r^2}\right)^2 - \left(1 - \frac{a^2}{r^2}\right)^2 \right] \quad (2)$$

$$\text{From Eq. 2: } \sin\theta \cos\theta \cdot 4a^2/r^2 = 0$$

$$\text{If } a \neq 0, r \neq 0 \text{ then } \sin\theta \cos\theta = 0 \text{ for which } \theta = 0, \frac{\pi}{2} \quad (3)$$

$$\text{Subst. into Eq. 1} \quad \theta = 0, 1 - a^2/r^2 = 0$$

$$\text{Giving } a = r$$

$$\text{For } \theta = \frac{\pi}{2} \quad 1 + a^2/r^2 = 0 \sim \text{impossible}$$

$$\text{Thus conditions for } |\nabla \psi|_{\max} \text{ are } \theta = 0 \quad r = a$$

1.17

$$P = P_o + \frac{1}{2} \rho v_\infty^2 \left[\frac{2xyz}{L^3} + 3 \left(\frac{x}{L} \right)^2 + \frac{v_\infty t}{L} \right]$$

$$\frac{dP}{dx} \vec{e}_x = \frac{1}{2} \rho v_\infty^2 \left[\frac{2yz}{L^3} + \frac{6x}{L^2} \right] \vec{e}_x$$

$$\frac{dP}{dy} \vec{e}_y = + \frac{1}{2} \rho v_\infty^2 \left[\frac{2xz}{L^3} \right] \vec{e}_y$$

$$\frac{dP}{dz} \vec{e}_z = + \frac{1}{2} \rho v_\infty^2 \left[\frac{2xy}{L^3} \right] \vec{e}_z$$

$$\nabla P = \frac{1}{2} \rho v_\infty^2 \left[\left(\frac{2yz}{L^3} + \frac{6x}{L^2} \right) \vec{e}_x + \frac{2xz}{L^3} \vec{e}_y + \frac{2xy}{L^3} \vec{e}_z \right]$$

1.18

Vertical cylinder $d=10\text{m}$, $h=6\text{m}$

$$V = \frac{\pi}{4} (10\text{m})^2 (6\text{m}) = 471.2 \text{ m}^3$$

@ 20°C $\rho_w = 998.2 \text{ kg/m}^3$

$$m = \rho_w V = (998.2)(471.2) = 470350 \text{ kg}$$

@ 80°C $\rho_w = 971.8 \text{ kg/m}^3$

$$m = (971.8)(471.2) = 457910 \text{ kg}$$

$$\Delta m = 12440 \text{ kg}$$

1.19

Liquid $V = 1200 \text{ cm}^3 @ 1.25 \text{ MPa}$
 $V = 1188 \text{ cm}^3 @ 2.5 \text{ MPa}$

$$\beta = -V \left(\frac{dV}{dP} \right)_T \cong -V \frac{\Delta V}{\Delta P}$$

$$V = 1194 \text{ cm}^3 = 1.194 \times 10^{-3} \text{ m}^3$$

$$\Delta V = -12 \text{ cm}^3 = -1.2 \times 10^{-7} \text{ m}^3$$

$$\beta = -1.194 \times 10^{-3} \left[\frac{1.25 \text{ MPa}}{-1.2 \times 10^{-7}} \right]$$

$$= +12440 \text{ MPa}$$

1.20

$$\beta = -V \left(\frac{dP}{dV} \right)_T \cong -V \frac{\Delta P}{\Delta V}$$

$$V = 0.25 \text{ m}^3$$

$$\Delta V = -0.005 \text{ m}^3$$

$$\Delta P = 10 \text{ mPa}$$

$$\beta = -0.25 \left[\frac{10}{-0.005} \right] = 500 \text{ MPa}$$

1.21

For H₂O: $\beta = 2.205 \text{ GPa}$

$$\frac{\Delta V}{V} = -0.0075$$

$$\beta \cong -V \frac{\Delta P}{\Delta V} \text{ or } \Delta P = \beta \frac{\Delta V}{V}$$

$$\Delta P = (2.205 \text{ GPa})(0.0075) = 0.0165 \text{ GPa} = 16.5 \text{ MPa}$$

1.22

For H₂O: P₁=100kPa P₂=120 MPa β = 2.205

$$\beta = -V \frac{\Delta P}{\Delta V} \quad \text{or} \quad \frac{\Delta V}{V} = \frac{\Delta P}{\beta}$$

$$\frac{\Delta V}{V} = \frac{\Delta P}{\beta} = \frac{(120000-100)kPa}{120 \times 10^6 kPa} = 0.999 \times 10^{-3} = 0.0999 \text{ percent}$$

1.23

H₂O@ 68°C (341 K)

$$\sigma = 0.123 [1 - 0.00139(341)] = 0.0647 \text{ N/m}$$

In a clean tube- $\theta = 0^\circ$

$$h = \frac{2\sigma \cos \theta}{\rho g r} = \frac{2(.0647)}{979(981) \left(\frac{0.2875 \times 10^{-2}}{2} \right)} = 9.37 \times 10^{-3} \text{ m} = 9.37 \text{ mm}$$

1.24

Parallel Glass Plates

Gap=1.625 mm

$\sigma = 0.0735$ N/m

For a unit depth:

Surface Tension Force = $2(1) \sigma \cos \theta$

Weight of H₂O = $\rho gh(1)(1.625 \times 10^{-3})$

For clean glass $\cos \theta = 1$

Equating Forces:

$2(1) \sigma = \rho gh(1)(1.625 \times 10^{-3})$

$h = \frac{2(0.0735)}{1000(9.81)(1.625 \times 10^{-3})} = 0.00922\text{m} = 9.22 \text{ mm}$

1.25

H₂O-Air-Glass Interface @40°C

Tube Radius= 1 mm

$$h = \frac{2\sigma \cos \theta}{\rho g r} \quad \cos \theta = 1$$

$$\sigma = 0.123[1 - 0.0139(313)] = 0.0695 \text{ N/m}$$

$$h = \frac{2(0.0695)}{993(9.81)(1 \times 10^{-3})} = 0.0143 \text{ m (1.43 cm)}$$

1.26

Soap Bubble- $T=20^{\circ}\text{C}$ $d=4\text{mm}$
 $\sigma=0.025\text{ N/m}$ (Table 1.2)

Force Balance for Bubble:

$$\pi r^2 \Delta P = 2\pi r \sigma$$

$$\text{so } \Delta P = \frac{2\sigma}{r} = \frac{2(0.025)}{2 \times 10^{-3}} = 25\text{ N/m}^2 = 25\text{ Pa}$$

1.27

@60°C

$$\sigma_{H_2O} = 0.0662 \text{ N/m}$$
$$\sigma_{Hg} = 0.44 \text{ N/m}$$

Tube Diameter = 0.55 mm

$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

For H₂O:

$$h = \frac{2(0.0662)\cos(0)}{983(9.81)\left(\frac{0.55 \times 10^{-3}}{2}\right)} = 0.0499 \text{ m (4.99 cm Rise)}$$

For Hg:

$$h = \frac{2(0.44)(\cos 130^\circ)}{13.6(983)(9.81)\left(\frac{0.55 \times 10^{-3}}{2}\right)} = -0.0157 \text{ m (1.57 cm Depression)}$$

1.28

H₂O/ Glass Interface

T=30°C

$$\sigma = 0.123[1 - 0.0139(303)] = 0.0712 \text{ N/m}$$

$$\rho = 996 \text{ kg/m}^3$$

$$h \leq 1 \text{ mm}$$

$$h = \frac{2\sigma \cos\theta}{\rho g r} \quad \cos\theta = 1$$

$$r = \frac{2\sigma}{\rho g h} = \frac{2(0.0712)}{996(9.81)(1 \times 10^{-3})} = 0.0146 \text{ m (1.457 cm)}$$

$$d = 2r = 2.915 \text{ cm}$$

1.29

Bubble Diameter = 0.25 cm = 0.0025 m, and so Radius = 0.00125 m

Capillary Tube: Diameter = 0.2cm = 0.002 m, and so Radius = 0.001 m

Beginning with:

$$\Delta P = \frac{2\sigma}{R}$$

Rearrange and remember the unit conversion Pa=kg/ms²,

$$\sigma = \frac{(P - P_0)R}{2} = \frac{(101453 \text{ Pa} - 101325 \text{ Pa})(0.00125 \text{ m})}{2} = 0.08 \text{ Pa} \cdot \text{m} = 0.08 \text{ kg/s}^2$$

Next, we can calculate the height of the fluid in the tube:

$$h = \frac{2\sigma \cos\theta}{\rho g r} = \frac{2(0.08 \text{ kg/s}^2) \cos(30)}{\left(750 \frac{\text{kg}}{\text{m}^3}\right) \left(9.8 \frac{\text{m}}{\text{s}}\right) (0.001 \text{ m})} = 0.01885 \text{ m} = \mathbf{1.885 \text{ cm}}$$

1.30

First, calculate the surface tension of water using the temperature of the water:

$$T = 80^{\circ}\text{C} = 353\text{K}$$

$$\sigma = 0.123(1 - 0.00139T)$$

$$\sigma = 0.123(1 - 0.00139(353)) = 0.06265 \text{ N/m}$$

Next, using the equation for the height of a fluid in a capillary,

$$h = \frac{2\sigma\cos\theta}{\rho g r}$$

Rearranging and solving for the radius:

$$r = \frac{2\sigma\cos\theta}{\rho g h}$$

$$\begin{aligned} &= \frac{2(0.06265 \text{ N/m}) \cos(0)}{\left(\left(\frac{97.18 \text{ g}}{100 \text{ mls}}\right) \left(\frac{\text{kg}}{1000\text{g}}\right) \left(\frac{1000 \text{ ml}}{\text{liter}}\right) \left(\frac{28.32 \text{ l}}{0.028317 \text{ m}^3}\right)\right) (9.81 \text{ m/s}^2)(17.5\text{mm}) \left(\frac{\text{m}}{1000\text{mm}}\right)} \\ &= 7.509 \times 10^{-4} \text{ meters} = 0.7509 \text{ mm} \end{aligned}$$

Diameter is 2r so D=1.50 mm