PROBLEM 2-1

GIVEN: Air with a molar composition of 79% N_2 and 21% O_2

FIND: The mass fraction of O_2 and N_2 in the air

APPROACH: This a simple conversion problem using:

$$MW_{mix} = \Sigma \chi_i MW_i \quad \text{and} \quad Y_i = \chi_i \frac{MW_i}{MW_{mix}}$$

Given the molar composition: $\chi_{N_2} = 0.79$ and $\chi_{O_2} = 0.29$

mixture molecular weight:

$$MW_{mix} = \Sigma \chi_i MW_i = \chi_{N_2} MW_{N_2} + \chi_{O_2} MW_{O_2}$$

$$= 0.79(28.013) + (0.21)(32) = 28.85 \text{ kg/kmol}$$

mass fraction of O_2 and N_2

$$Y_{O_2} = \chi_{O_2}\left(\frac{MW_{O_2}}{MW_{mix}}\right) = 0.21\left(\frac{32}{28.85}\right) = 0.233$$

$$Y_{N_2} = \chi_{N_2}\left(\frac{MW_{N_2}}{MW_{mix}}\right) = 0.79\left(\frac{28.013}{28.85}\right) = 0.767$$

Comments: Note that $Y_{O_2} > \chi_{O_2}$ since $MW_{O_2} > MW_{mix}$ and that $\Sigma Y_i = 1$ as would be expected.

PROBLEM 2-2

GIVEN: The following mixture:

Species	# MOLES	X_i	Y_i	MW_i
CO	0.095	0.002	0.002	28.010
CO_2	6	0.127	0.195	44.011
H_2O	7	0.149	0.094	18.016
N_2	34	0.722	0.707	28.013
NO	0.005	106×10^{-6}	111×10^{-6}	30.006
TOTAL:	47.1	1.0	1.0	

FIND: a) The mole fraction, mole %, and ppm of NO in the mixture
b) Determine the MW of the mixture
c) Determine the mass fraction of each constituent

a) $$\chi_i = \frac{N_i}{\Sigma N_i} = \frac{N_{NO}}{N_{CO}+N_{CO_2}+N_{H_2O}+N_{N_2}+N_{NO}} = \frac{0.005}{0.095+6+7+34+0.005}$$

$$\boxed{\chi_{NO} = 106\times10^{-6}\ \text{Kmol}/\text{Kmol-mix}}$$

$$\boxed{\text{MOLE }\% = \chi_i \cdot 100 = 0.0106\ \%}$$

$$PPM = \frac{\#NO}{TOT\ \#}(1\times10^6) = \frac{N_{NO}\cdot A}{\Sigma N_i\cdot A}(1\times10^6) = \chi_{NO}(1\times10^6) = \boxed{106\ ppm}$$

where $A \equiv$ Avogadro's Number

b) $$MW_{mix} = \Sigma \chi_i MW_i = \chi_{CO}MW_{CO} + \chi_{CO_2}MW_{CO_2} + \chi_{H_2O}MW_{H_2O} + \chi_{N_2}MW_{N_2} + \chi_{NO}MW_{NO}$$

where χ_{CO}, χ_{CO_2}, χ_{H_2O}, and χ_{N_2} are found in the same manner as χ_{NO} was found

$$MW_{MIX} = (0.002)(28.010) + (0.127)(44.011) + (0.149)(18.016) + 0.722(28.013) + (106\times10^{-6})(33.006)$$

$$\boxed{MW_{MIX} = 28.6\ \text{kg}/\text{Kmol-mix}}$$

c) $$Y_i = \chi_i\frac{MW_i}{MW_{MIX}} \longrightarrow$$ (Kg_i/Kg_{mix})

CO: $Y = (0.002)(28.01/28.6) = 0.002$
CO_2: $Y = (0.127)(44.011/28.6) = 0.195$
H_2O: $Y = (0.149)(18.016/28.6) = 0.094$
N_2: $Y = (0.722)(28.013/28.6) = 0.707$
NO: $Y = (106\times10^{-6})(30.006/28.6) = 111\times10^{-6}$

COMMENTS: Note that $ppm = \chi_i(1\times10^6)$ and that $\Sigma\chi_i = 1$ and $\Sigma Y_i = 1$ can often be used to check your calculations

PROBLEM 2-3

GIVEN: mixture with 5 kmole H_2 and 3 kmole O_2

FIND: χ_{H_2}, χ_{O_2}, MW_{mix}, Y_{H_2}, Y_{O_2}

SOLUTION:

a) $\chi_i = \dfrac{N_i}{N_{tot}}$; $\chi_{H_2} = \dfrac{5}{5+3} = \boxed{0.625}$

$\chi_{O_2} = 1 - \chi_{H_2} = 1 - 0.625 = \boxed{0.375}$

b) $MW_{mix} = \Sigma \chi_i MW_i = \chi_{H_2} MW_{H_2} + \chi_{O_2} MW_{O_2}$

$= 0.625 (2.016) + .375 (31.999)$

$\boxed{MW_{mix} = 13.260}$

c) $Y_i = \chi_i \dfrac{MW_i}{MW_{mix}}$; $Y_{H_2} = 0.625 \dfrac{2.016}{13.260} = \boxed{0.095}$

$Y_{O_2} = 1 - Y_{H_2} = 1 - 0.095 = \boxed{0.905}$

COMMENT: Even though the mole fraction of H_2 is large, its low molecular weight results in its having a small mass fraction.

PROBLEM 2-4

GIVEN: $O_2 - CH_4$ mixture @ 300 K & 100 kPa; $\chi_{O_2} = 0.2$

FIND: Y_{CH_4}, N_{CH_4}/V

ASSUMPTIONS: ideal gas mixture

SOLUTION:

a) $$Y_{CH_4} = \chi_{CH_4}\frac{MW_{CH_4}}{MW_{MIX}}$$

$$= \chi_{CH_4}\frac{MW_{CH_4}}{\chi_{CH_4} MW_{CH_4} + (1-\chi_{CH_4}) MW_{O_2}}$$

$$= 0.2\,\frac{16.043}{0.2(16.043) + 0.8(31.999)} = \frac{0.2(16.043)}{28.808}$$

$$\boxed{Y_{CH_4} = 0.111}$$

b) $$P_{CH_4} V = N_{CH_4} R_u T \; ; \quad P_{CH_4} = \chi_{CH_4} P$$

$$N_{CH_4}/V = \frac{\chi_{CH_4} P}{R_u T}$$

$$= \frac{(0.2)\,100 \cdot 10^3}{8315\,(300)} = \boxed{8.018 \cdot 10^{-3}\ \frac{kmol}{m^3}}$$

COMMENT: Careful treatment of units is required in part b.

PROBLEM 2-5

GIVEN: $N_2 - Ar$ mixture with $N_{N_2} = 3 N_{Ar}$;
$T = 500\ K$; $P = 250\ kPa$

FIND: χ_i, MW_{mix}, Y_i, N_{N_2}/V

ASSUMPTION: ideal gas mixture

SOLUTION:

a) $$\chi_{N_2} = \frac{N_{N_2}}{N_{MIX}} = \frac{3 N_{Ar}}{3N_{Ar} + N_{Ar}} = \frac{3}{4} = \boxed{0.75}$$

$$\chi_{Ar} = 1 - 0.75 = \boxed{0.25}$$

b) $$MW_{mix} = \sum \chi_i MW_i$$
$$= 0.75(28.014) + 0.25(39.948)$$
$$\boxed{MW_{mix} = 30.998}$$

c) $$Y_{N_2} = \chi_{N_2} \frac{MW_{N_2}}{MW_{mix}} = 0.75 \frac{28.014}{30.998} = \boxed{0.678}$$
$$\boxed{Y_{Ar} = 1 - Y_{N_2} = 0.322}$$

d) $P_{N_2} V = N_{N_2} R_u T$; $P_{N_2} = \chi_{N_2} P_{tot}$

$$N_{N_2}/V = \frac{\chi_{N_2} P_{tot}}{R_u T} = \frac{(0.75)\,250 \cdot 10^3}{8315\,(500)} = \boxed{0.0451\ kmol_{N_2}/m^3}$$

COMMENT: Careful treatment of units is required in part d.

PROBLEM 2-6

GIVEN: $CO_2 - O_2$ mixture with $\chi_{CO_2} = 0.1$; $\chi_{O_2} = 0.9$; $T = 400\,K$

FIND: Standardized enthalpy of mixture

ASSUMPTION: Ideal-gas behavior

SOLUTION: This is a straightforward application of Eqn. 2.15a combined with the definition of standardized enthalpy (Eqn. 2.34).

$$\bar{h}_{CO_2} = \bar{h}^o_{f,CO_2} + \Delta\bar{h}_{s,CO_2} = -393{,}546 + 4003 \quad \text{(Table A.2)}$$
$$= -389{,}543 \text{ kJ/kmol}$$

$$\bar{h}_{O_2} = \bar{h}^o_{f,O_2} + \Delta\bar{h}_{s,O_2} = 0 + 3031 = 3031 \text{ kJ/kmol}$$
(Table A.11)

$$\bar{h}_{mix} = \sum \chi_i \bar{h}_i = \chi_{CO_2}\bar{h}_{CO_2} + \chi_{O_2}\bar{h}_{O_2} \quad \text{(Eqn. 2.15a)}$$
$$= 0.1(-389{,}543) + 0.9(3031)$$

$$\boxed{\bar{h}_{mix} = -36{,}226 \text{ kJ/kmol}_{mix}}$$

COMMENT: The use of the Appendix A tables made this problem simple. Note that the same information is available as curvefit equations in Table A-13.