برای دسترسی به نسخه کامل حل المسائل، روی لینک زیر کلیک کنید و یا به وبسایت "ایبوک یاب" مراجعه بفرمایید
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CHAPTER ONE

Solutions for Section 1.1

Exercises

1. Since t represents the number of years since 2010, we see that $f(5)$ represents the population of the city in 2015. In 2015, the city's population was 7 million.
2. Since $T = f(P)$, we see that $f(200)$ is the value of T when $P = 200$; that is, the thickness of pelican eggs when the concentration of PCBs is 200 ppm.
3. If there are no workers, there is no productivity, so the graph goes through the origin. At first, as the number of workers increases, productivity also increases. As a result, the curve goes up initially. At a certain point the curve reaches its highest level, after which it goes downward; in other words, as the number of workers increases beyond that point, productivity decreases. This might, for example, be due either to the inefficiency inherent in large organizations or simply to workers getting in each other's way as too many are crammed on the same line. Many other reasons are possible.
4. The slope is $(1-0)/(1-0) = 1$. So the equation of the line is $y = x$.
5. The slope is $(3-2)/(2-0) = 1/2$. So the equation of the line is $y = (1/2)x + 2$.
6. The slope is
$$\text{Slope } = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2}.$$
Now we know that $y = (1/2)x + b$. Using the point $(-2, 1)$, we have $1 = -2/2 + b$, which yields $b = 2$. Thus, the equation of the line is $y = (1/2)x + 2$.
7. The slope is $\dfrac{6-0}{2-(-1)} = 2$ so the equation of the line is $y - 6 = 2(x-2)$ or $y = 2x + 2$.
8. Rewriting the equation as $y = -\frac{5}{2}x + 4$ shows that the slope is $-\frac{5}{2}$ and the vertical intercept is 4.
9. Rewriting the equation as
$$y = -\frac{12}{7}x + \frac{2}{7}$$
shows that the line has slope $-12/7$ and vertical intercept $2/7$.
10. Rewriting the equation of the line as
$$-y = \frac{-2}{4}x - 2$$
$$y = \frac{1}{2}x + 2,$$
we see the line has slope $1/2$ and vertical intercept 2.
11. Rewriting the equation of the line as
$$y = \frac{12}{6}x - \frac{4}{6}$$
$$y = 2x - \frac{2}{3},$$
we see that the line has slope 2 and vertical intercept $-2/3$.
12. (a) is (V), because slope is positive, vertical intercept is negative
(b) is (IV), because slope is negative, vertical intercept is positive
(c) is (I), because slope is 0, vertical intercept is positive
(d) is (VI), because slope and vertical intercept are both negative
(e) is (II), because slope and vertical intercept are both positive
(f) is (III), because slope is positive, vertical intercept is 0

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13. (a) is (V), because slope is negative, vertical intercept is 0
(b) is (VI), because slope and vertical intercept are both positive
(c) is (I), because slope is negative, vertical intercept is positive
(d) is (IV), because slope is positive, vertical intercept is negative
(e) is (III), because slope and vertical intercept are both negative
(f) is (II), because slope is positive, vertical intercept is 0

14. The intercepts appear to be $(0, 3)$ and $(7.5, 0)$, giving

$$\text{Slope } = \frac{-3}{7.5} = -\frac{6}{15} = -\frac{2}{5}.$$

The y-intercept is at $(0, 3)$, so a possible equation for the line is

$$y = -\frac{2}{5}x + 3.$$

(Answers may vary.)

15. $y - c = m(x - a)$

16. Given that the function is linear, choose any two points, for example $(5.2, 27.8)$ and $(5.3, 29.2)$. Then

$$\text{Slope} = \frac{29.2 - 27.8}{5.3 - 5.2} = \frac{1.4}{0.1} = 14.$$

Using the point-slope formula, with the point $(5.2, 27.8)$, we get the equation

$$y - 27.8 = 14(x - 5.2)$$

which is equivalent to

$$y = 14x - 45.$$

17. $y = 5x - 3$. Since the slope of this line is 5, we want a line with slope $-\frac{1}{5}$ passing through the point $(2, 1)$. The equation is $(y - 1) = -\frac{1}{5}(x - 2)$, or $y = -\frac{1}{5}x + \frac{7}{5}$.

18. The line $y + 4x = 7$ has slope -4. Therefore the parallel line has slope -4 and equation $y - 5 = -4(x - 1)$ or $y = -4x + 9$. The perpendicular line has slope $\frac{-1}{(-4)} = \frac{1}{4}$ and equation $y - 5 = \frac{1}{4}(x - 1)$ or $y = 0.25x + 4.75$.

19. The line parallel to $y = mx + c$ also has slope m, so its equation is

$$y = m(x - a) + b.$$

The line perpendicular to $y = mx + c$ has slope $-1/m$, so its equation will be

$$y = -\frac{1}{m}(x - a) + b.$$

20. Since the function goes from $x = 0$ to $x = 4$ and between $y = 0$ and $y = 2$, the domain is $0 \le x \le 4$ and the range is $0 \le y \le 2$.

21. Since x goes from 1 to 5 and y goes from 1 to 6, the domain is $1 \le x \le 5$ and the range is $1 \le y \le 6$.

22. Since the function goes from $x = -2$ to $x = 2$ and from $y = -2$ to $y = 2$, the domain is $-2 \le x \le 2$ and the range is $-2 \le y \le 2$.

23. Since the function goes from $x = 0$ to $x = 5$ and between $y = 0$ and $y = 4$, the domain is $0 \le x \le 5$ and the range is $0 \le y \le 4$.

24. The domain is all numbers. The range is all numbers ≥ 2, since $x^2 \ge 0$ for all x.

25. The domain is all x-values, as the denominator is never zero. The range is $0 < y \le \frac{1}{2}$.

26. The value of $f(t)$ is real provided $t^2 - 16 \ge 0$ or $t^2 \ge 16$. This occurs when either $t \ge 4$, or $t \le -4$. Solving $f(t) = 3$, we have

$$\begin{aligned} \sqrt{t^2 - 16} &= 3 \\ t^2 - 16 &= 9 \\ t^2 &= 25 \end{aligned}$$

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so

$$t = \pm 5.$$

27. We have $V = kr^3$. You may know that $V = \frac{4}{3}\pi r^3$.

28. If distance is d, then $v = \frac{d}{t}$.

29. For some constant k, we have $S = kh^2$.

30. We know that E is proportional to v^3, so $E = kv^3$, for some constant k.

31. We know that N is proportional to $1/l^2$, so

$$N = \frac{k}{l^2}, \quad \text{for some constant } k.$$

Problems

32. (a) Each date, t, has a unique daily snowfall, S, associated with it. So snowfall is a function of date.
(b) On December 12, the snowfall was approximately 5 inches.
(c) On December 11, the snowfall was above 10 inches.
(d) Looking at the graph we see that the largest increase in the snowfall was between December 10 to December 11.

33. (a) When the car is 5 years old, it is worth $6000.
(b) Since the value of the car decreases as the car gets older, this is a decreasing function. A possible graph is in Figure 1.1:

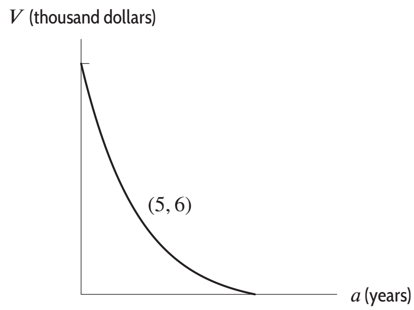

Figure 1.1

(c) The vertical intercept is the value of V when $a = 0$, or the value of the car when it is new. The horizontal intercept is the value of a when $V = 0$, or the age of the car when it is worth nothing.

34. (a) The story in (a) matches Graph (IV), in which the person forgot her books and had to return home.
(b) The story in (b) matches Graph (II), the flat tire story. Note the long period of time during which the distance from home did not change (the horizontal part).
(c) The story in (c) matches Graph (III), in which the person started calmly but sped up later.

The first graph (I) does not match any of the given stories. In this picture, the person keeps going away from home, but his speed decreases as time passes. So a story for this might be: *I started walking to school at a good pace, but since I stayed up all night studying calculus, I got more and more tired the farther I walked.*

35. The year 2000 was 12 years before 2012 so 2000 corresponds to $t = 12$. Thus, an expression that represents the statement is:

$$f(12) = 7.049.$$

36. The year 2012 was 0 years before 2012 so 2012 corresponds to $t = 0$. Thus, an expression that represents the statement is:

$$f(0) \text{ meters}.$$

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37. The year 1949 was $2012 - 1949 = 63$ years before 2012 so 1949 corresponds to $t = 63$. Similarly, we see that the year 2000 corresponds to $t = 12$. Thus, an expression that represents the statement is:

$$f(63) = f(12).$$

38. Since $t = 1$ means one year before 2012, then $t = 1$ corresponds to the year 2011. Similarly, $t = 0$ corresponds to the year 2012. Thus, $f(1)$ and $f(0)$ are the average annual sea level values, in meters, in 2011 and 2012, respectively. Because 8 millimeters is the same as 0.008 meters, the average sea level in 2012, $f(0)$, is 0.008 less than the sea level in 2011 which is $f(1)$. An expression that represents the statement is:

$$f(0) = f(1) - 0.008.$$

Note that there are other possible equivalent expressions, such as: $f(0) - f(1) = -0.008$.

39. See Figure 1.2.

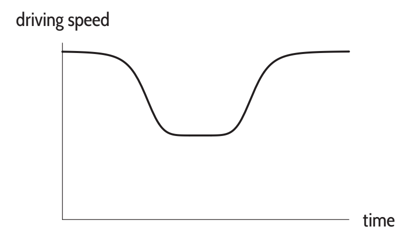

Figure 1.2

40. See Figure 1.3.

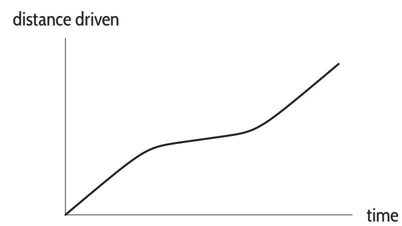

Figure 1.3

41. See Figure 1.4.

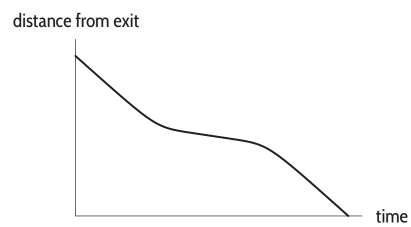

Figure 1.4

42. See Figure 1.5.

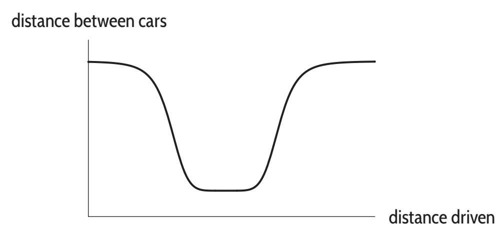

Figure 1.5

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43. (a) $f(30) = 10$ means that the value of f at $t = 30$ was 10. In other words, the temperature at time $t = 30$ minutes was 10°C. So, 30 minutes after the object was placed outside, it had cooled to 10 °C.

(b) The intercept a measures the value of $f(t)$ when $t = 0$. In other words, when the object was initially put outside, it had a temperature of a°C. The intercept b measures the value of t when $f(t) = 0$. In other words, at time b the object's temperature is 0 °C.

44. (a) The height of the rock decreases as time passes, so the graph falls as you move from left to right. One possibility is shown in Figure 1.6.

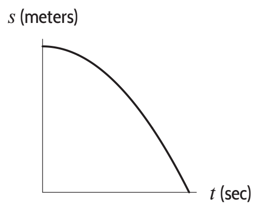

Figure 1.6

(b) The statement $f(7) = 12$ tells us that 7 seconds after the rock is dropped, it is 12 meters above the ground.

(c) The vertical intercept is the value of s when $t = 0$; that is, the height from which the rock is dropped. The horizontal intercept is the value of t when $s = 0$; that is, the time it takes for the rock to hit the ground.

45. See Figure 1.7.

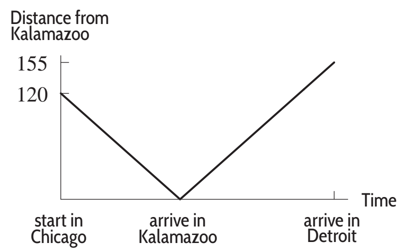

Figure 1.7

46. (a) We select two points on the line. Using the values (1992, 6.75) and (2006, 12.25) we have

$$\text{Slope} = \frac{12.25 - 6.75}{2006 - 1992 \text{ years}} = 0.4 \frac{\text{million barrels per day}}{\text{year}}.$$

(b) With t as the year and $f(t)$ as the quantity of imports in millions of barrels per day, we have

$$f(t) = 6.75 + 0.4(t - 1992).$$

Alternatively, if t as the number of years since 1992 and if $f(t)$ is the quantity of imports in millions of barrels per day

$$f(t) = 6.75 + 0.4t.$$

(c) We have

$$6.75 + 0.4t = 18$$
$$t = 28.125.$$

The model predicts imports will reach 18 million barrels a day in the year $1992 + 28.125 = 2020.125$.

This prediction could serve as a guideline, but it is very risky to put too much reliance on a prediction more than ten years into the future. Many unexpected events could drastically change the economic environment during that time. This prediction should be used with caution.

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47. (a) Reading coordinates from the graph, we see that rainfall $r = 100$ mm corresponds to about $Q = 600$ kg/hectare, and $r = 600$ mm corresponds to about $Q = 5800$ kg/hectare. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta Q}{\Delta r} = \frac{5800 - 600}{600 - 100} = 10.4 \text{ kg/hectare per mm.}$$

(b) Every additional 1 mm of annual rainfall corresponds to an additional 10.4 kg of grass per hectare.

(c) Using the slope, we see that the equation has the form

$$Q = b + 10.4r.$$

Substituting $r = 100$ and $Q = 600$ we can solve for b.

$$\begin{aligned} b + 10.4(100) &= 600 \\ b &= -440. \end{aligned}$$

The equation of the line is

$$Q = -440 + 10.4r.$$

48. (a) Reading coordinates from the graph, we see that rainfall $r = 100$ mm corresponds to about $Q = 300$ kg/hectare, and $r = 600$ mm corresponds to about $Q = 3200$ kg/hectare. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta Q}{\Delta r} = \frac{3200 - 300}{600 - 100} = 5.8 \text{ kg/hectare per mm.}$$

(b) Every additional 1 mm of annual rainfall corresponds to an additional 5.8 kg of grass per hectare.

(c) Using the slope, we see that the equation has the form

$$Q = b + 5.8r.$$

Substituting $r = 100$ and $Q = 300$ we can solve for b.

$$\begin{aligned} b + 5.8(100) &= 300 \\ b &= -280. \end{aligned}$$

The equation of the line is

$$Q = -280 + 5.8r.$$

49. The difference quotient $\Delta Q/\Delta r$ equals the slope of the line and represents the increase in the quantity of grass per millimeter of rainfall. We see from the graph that the slope of the line for 1939 is larger than the slope of the line for 1997. Thus, each additional 1 mm of rainfall in 1939 led to a larger increase in the quantity of grass than in 1997.

50. (a) Let x be the average minimum daily temperature (°C), and let y be the date the marmot is first sighted. Reading coordinates from the graph, we see that temperature $x = 12$°C corresponds to date $y = 137$ days after Jan 1, and $x = 22$°C corresponds to $y = 109$ days. Using a difference quotient, we have

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{109 - 137}{22 - 12} = -2.8 \frac{\text{days}}{°\text{C}}.$$

(b) The slope is negative. An increase in the temperatures corresponds to an earlier sighting of a marmot. Marmots come out of hibernation earlier in years with warmer average daily minimum temperature.

(c) We have

$$\Delta y = \text{Slope} \times \Delta x = (-2.8)(6) = 16.8 \text{ days.}$$

If temperatures are 6°C higher, then marmots come out of hibernations about 17 days earlier.

(d) Using the slope, we see that the equation has the form

$$y = b - 2.8x.$$

Substituting $x = 12$ and $y = 137$ we solve for b.

$$\begin{aligned} b - 2.8(12) &= 137 \\ b &= 171. \end{aligned}$$

The equation of the line is

$$y = 171 - 2.8x.$$

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51. (a) If the first date of bare ground is 140, then, according to the figure, the first bluebell flower is sighted about day 150, that is $150 - 140 = 10$ days later.

(b) Let x be the first date of bare ground, and let y be the date the first bluebell flower is sighted. Reading coordinates from the graph, we see that date $x = 130$ days after Jan 1 corresponds to date $y = 142$ days after Jan 1, and $x = 170$ days corresponds to $y = 173$ days. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{173 - 142}{170 - 130} = 0.775 \text{ days per day.}$$

(c) The slope is positive, so an increase in the x-variable corresponds to an increase in the y-variable. These variables represents days in the year, and larger values indicate days that are later in the year. Thus if bare ground first occurs later in the year, then bluebells first flower later in the year. Positive slope means that bluebells flower later when the snow cover lasts longer.

(d) Using the slope, we see that the equation has the form

$$y = b + 0.775x.$$

Substituting $x = 130$ and $y = 142$ we can solve for b.

$$\begin{aligned} b + 0.775(130) &= 142 \\ b &= 41.25. \end{aligned}$$

The equation of the line is

$$y = 41.25 + 0.775x.$$

52. (a) We have $m = \dfrac{256}{18} = 14.222$ cm per hour. When the snow started, there were 100 cm on the ground, so

$$f(t) = 100 + 14.222t.$$

(b) The domain of f is $0 \le t \le 18$ hours. The range is $100 \le f(t) \le 356$ cm.

53. (a) We find the slope m and intercept b in the linear equation $C = b + mw$. To find the slope m, we use

$$m = \frac{\Delta C}{\Delta w} = \frac{12.32 - 8}{68 - 32} = 0.12 \text{ dollars per gallon.}$$

We substitute to find b:

$$\begin{aligned} C &= b + mw \\ 8 &= b + (0.12)(32) \\ b &= 4.16 \text{ dollars.} \end{aligned}$$

The linear formula is $C = 4.16 + 0.12w$.

(b) The slope is 0.12 dollars per gallon. Each additional gallon of waste collected costs 12 cents.

(c) The intercept is \$4.16. The flat monthly fee to subscribe to the waste collection service is \$4.16. This is the amount charged even if there is no waste.

54. We are looking for a linear function $y = f(x)$ that, given a time x in years, gives a value y in dollars for the value of the refrigerator. We know that when $x = 0$, that is, when the refrigerator is new, $y = 950$, and when $x = 7$, the refrigerator is worthless, so $y = 0$. Thus $(0, 950)$ and $(7, 0)$ are on the line that we are looking for. The slope is then given by

$$m = \frac{950}{-7}$$

It is negative, indicating that the value decreases as time passes. Having found the slope, we can take the point $(7, 0)$ and use the point-slope formula:

$$y - y_1 = m(x - x_1).$$

So,

$$\begin{aligned} y - 0 &= -\frac{950}{7}(x - 7) \\ y &= -\frac{950}{7}x + 950. \end{aligned}$$

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55. (a) Charge per cubic foot $= \dfrac{\Delta \$}{\Delta \text{ cu. ft.}} = \dfrac{55-40}{1600-1000} = \$0.025/\text{cu. ft.}$
Alternatively, if we let c = cost, w = cubic feet of water, b = fixed charge, and m = cost/cubic feet, we obtain $c = b + mw$. Substituting the information given in the problem, we have

$$40 = b + 1000m$$
$$55 = b + 1600m.$$

Subtracting the first equation from the second yields $15 = 600m$, so $m = 0.025$.

(b) The equation is $c = b + 0.025w$, so $40 = b + 0.025(1000)$, which yields $b = 15$. Thus the equation is $c = 15 + 0.025w$.

(c) We need to solve the equation $100 = 15 + 0.025w$, which yields $w = 3400$. It costs \$100 to use 3400 cubic feet of water.

56. (a) We find the slope m and intercept b in the linear equation $S = b + mt$. To find the slope m, we use

$$m = \frac{\Delta S}{\Delta t} = \frac{66-113}{50-0} = -0.94.$$

When $t = 0$, we have $S = 113$, so the intercept b is 113. The linear formula is

$$S = 113 - 0.94t.$$

(b) We use the formula $S = 113 - 0.94t$. When $S = 20$, we have $20 = 113 - 0.94t$ and so $t = 98.9$. If this linear model were correct, the average male sperm count would drop below the fertility level during the year 2038.

57. (a) (i) $f(2001) = 272$
(ii) $f(2014) = 525$

(b) The average yearly increase is the rate of change.

$$\text{Yearly increase} = \frac{f(2014) - f(2001)}{2014 - 2001} = \frac{525 - 272}{13} = 19.46 \text{ billionaires per year.}$$

(c) Since we assume the rate of increase remains constant, we use a linear function with slope 19.46 billionaires per year. The equation is

$$f(t) = b + 19.46t$$

where $f(2001) = 272$, so

$$272 = b + 19.46(2001)$$
$$b = -38{,}667.5.$$

Thus, $f(t) = 19.46t - 38{,}667.5$.

58. (a) The variable costs for x acres are $\$200x$, or $0.2x$ thousand dollars. The total cost, C (again in thousands of dollars), of planting x acres is:

$$C = f(x) = 10 + 0.2x.$$

This is a linear function. See Figure 1.8. Since $C = f(x)$ increases with x, f is an increasing function of x. Look at the values of C shown in the table; you will see that each time x increases by 1, C increases by 0.2. Because C increases at a constant rate as x increases, the graph of C against x is a line.

(b) See Figure 1.8 and Table 1.1.

Table 1.1 *Cost of planting seed*

x	C
0	10
2	10.4
3	10.6
4	10.8
5	11
6	11.2

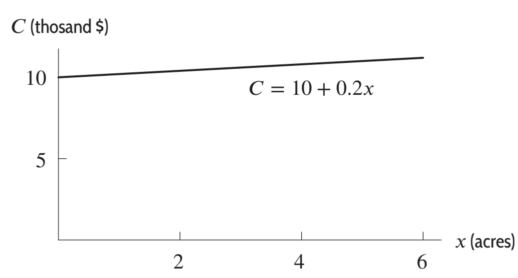

Figure 1.8

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(c) The vertical intercept of 10 corresponds to the fixed costs. For $C = f(x) = 10 + 0.2x$, the intercept on the vertical axis is 10 because $C = f(0) = 10 + 0.2(0) = 10$. Since 10 is the value of C when $x = 0$, we recognize it as the initial outlay for equipment, or the fixed cost.

The slope 0.2 corresponds to the variable costs. The slope is telling us that for every additional acre planted, the costs go up by 0.2 thousand dollars. The rate at which the cost is increasing is 0.2 thousand dollars per acre. Thus the variable costs are represented by the slope of the line $f(x) = 10 + 0.2x$.

59. We will let

$$\begin{aligned} T &= \text{amount of fuel for take-off,} \\ L &= \text{amount of fuel for landing,} \\ P &= \text{amount of fuel per mile in the air,} \\ m &= \text{the length of the trip in miles.} \end{aligned}$$

Then Q, the total amount of fuel needed, is given by

$$Q(m) = T + L + Pm.$$

60. (a) The scale of the graph makes it impossible to read values of $f(423)$ and $f(422)$ accurately enough to evaluate their difference. But that difference equals the slope of the line, which we can estimate. Using the points $(400, 2000)$ and $(600, 6000)$ on the line, we have

$$\text{Slope} = \frac{6000 - 2000}{600 - 400} = 20.$$

Thus $f(423) - f(422) = 20$.

(b) We have

$$\begin{aligned} \Delta y &= \text{Slope} \times \Delta x \\ &= (20)(517 - 513) = 80. \end{aligned}$$

Thus $f(517) - f(513) = 80$.

61. (a) The scale of the graph makes it impossible to read values of $g(4210)$ and $g(4209)$ accurately enough to evaluate their difference. But that difference equals the slope of the line, which we can estimate. Using the points $(3000, 70)$ and $(5000, 50)$ on the line, we have

$$\text{Slope} = \frac{50 - 70}{5000 - 3000} = -0.01.$$

So

$$\begin{aligned} \Delta y &= \text{Slope} \times \Delta x \\ &= (-0.01)(4210 - 4209) = -0.01. \end{aligned}$$

Thus $g(4210) - g(4209) = -0.01$.

(b) We have

$$\begin{aligned} \Delta y &= \text{Slope} \times \Delta x \\ &= (-0.01)(3760 - 3740) = -0.2. \end{aligned}$$

Thus $g(3760) - g(3740) = -0.2$.

62. (a) The largest time interval was 2010–2012 since the percentage growth rate increased from -60.5 to 69.9 from 2010 to 2012. This means the US exports of biofuels grew relatively more from 2010 to 2012 than from 2012 to 2013. (Note that the percentage growth rate was a decreasing function of time over 2012–2014.)

(b) The largest time interval was 2012–2013 since the percentage growth rates were positive for each of these two consecutive years. This means that the amount of biofuels exported from the US steadily increased during the two years from 2012 to 2013, then decreased in 2014.

63. (a) The largest time interval was 2007–2009 since the percentage growth rate increased from -14.6 in 2007 to 6.3 in 2009. This means that from 2007 to 2009 the US consumption of hydroelectric power grew relatively more with each successive year.

(b) The largest time interval was 2012–2014 since the percentage growth rates were negative for each of these three consecutive years. This means that the amount of hydroelectric power consumed by the US industrial sector steadily decreased during the three year span from 2012 to 2014.

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64. (a) The largest time interval was 2008–2010 since the percentage growth rate decreased from 3.6 in 2008 to −29.7 in 2010. This means that from 2008 to 2010 the US price per watt of a solar panel fell relatively more with each successive year.

(b) The largest time interval was 2009–2010 since the percentage growth rates were negative for each of these two consecutive years. This means that the US price per watt of a solar panel decreased during the two year span from 2009 to 2010, after an increase in the previous year.

65. (a) Since 2008 corresponds to $t = 0$, the average annual sea level in Aberdeen in 2008 was 7.094 meters.

(b) Looking at the table, we see that the average annual sea level was 7.019 twenty five years before 2008, or in the year 1983. Similar reasoning shows that the average sea level was 6.957 meters 125 years before 2008, or in 1883.

(c) Because 125 years before 2008 the year was 1883, we see that the sea level value corresponding to the year 1883 is 6.957 (this is the sea level value corresponding to $t = 125$). Similar reasoning yields the table:

Year	1883	1908	1933	1958	1983	2008
S	6.957	6.938	6.965	6.992	7.019	7.094

66. (a) This could be a linear function because w increases by 2.7 as h increases by 4.

(b) We find the slope m and the intercept b in the linear equation $w = b + mh$. We first find the slope m using the first two points in the table. Since we want w to be a function of h, we take

$$m = \frac{\Delta w}{\Delta h} = \frac{82.4 - 79.7}{172 - 168} = 0.68.$$

Substituting the first point and the slope $m = 0.68$ into the linear equation $w = b+mh$, we have $79.7 = b+(0.68)(168)$, so $b = -34.54$. The linear function is

$$w = 0.68h - 34.54.$$

The slope, $m = 0.68$, is in units of kg per cm.

(c) We find the slope and intercept in the linear function $h = b + mw$ using $m = \Delta h/\Delta w$ to obtain the linear function

$$h = 1.47w + 50.79.$$

Alternatively, we could solve the linear equation found in part (b) for h. The slope, $m = 1.47$, has units cm per kg.

67. (a) The first company's price for a day's rental with m miles on it is $C_1(m) = 40+0.15m$. Its competitor's price for a day's rental with m miles on it is $C_2(m) = 50 + 0.10m$.

(b) See Figure 1.9.

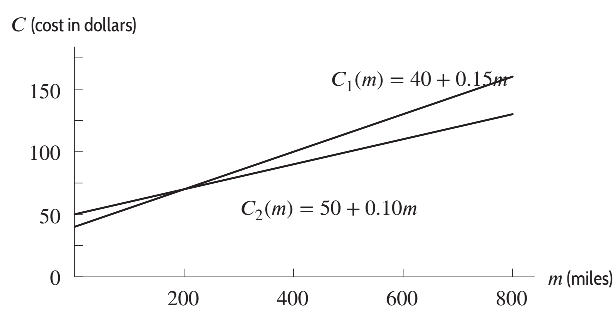

Figure 1.9

(c) To find which company is cheaper, we need to determine where the two lines intersect. We let $C_1 = C_2$, and thus

$$40 + 0.15m = 50 + 0.10m$$
$$0.05m = 10$$
$$m = 200.$$

If you are going more than 200 miles a day, the competitor is cheaper. If you are going less than 200 miles a day, the first company is cheaper.

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68. (a) Since the initial value is \$25,000 and the slope is −2000, the value of the vehicle at time t is

$$V(t) = 25000 - 2000t.$$

The cost of repairs has initial value 0 and slope 1500, so

$$C(t) = 1500t.$$

Figure 1.10 shows the graphs of these two functions.

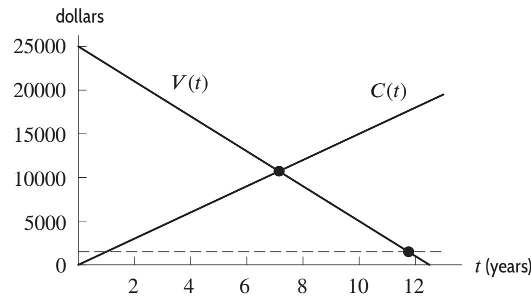

Figure 1.10

(b) The vehicle value equals the repair cost, when $V(t) = C(t)$.

$$\begin{aligned} 25000 - 2000t &= 1500t \\ 25000 &= 3500t \\ 7.143 &= t. \end{aligned}$$

Thus, replace during the eighth year. Since 0.143 years is $0.143(12) = 1.7$ months, replacement should take place in the second month of the eighth year.

(c) Since 6% of the orginal value is 1500, the vehicle should be replaced when $V(t) = 1500$.

$$\begin{aligned} 25000 - 2000t &= 1500 \\ 25000 &= 1500 + 2000t \\ 23500 &= 2000t \\ 11.750 &= t. \end{aligned}$$

Thus, in the twelfth year. Since 0.75 years is 9 months, replacement should take place at the end of the ninth month of the twelfth year.

69. (a) If the bakery owner decreases the price, the customers want to buy more. Thus, the slope of $d(q)$ is negative. If the owner increases the price, she is make more cakes. Thus the slope of $s(q)$ is positive.

(b) To determine whether an ordered pair (q, p) is a solution to the inequality, we substitute the values of q and p into the inequality and see whether the resulting statement is true. Substituting the two ordered pairs gives

(60, 18): $18 \le 20 - 60/20$, or $18 \le 17$. This is false, so (60, 18) is not a solution.

(120, 12): $12 \le 20 - 120/20$, or $12 \le 14$. This is true, so (120, 12) is a solution.

The pair (60, 18) is not a solution to the inequality $p \le 20 - q/20$. This means that the price \$18 is higher than the unit price at which customers would be willing to buy a total of 60 cakes. So customers are not willing to buy 60 cakes at \$18. The pair (120, 12) is a solution, meaning that \$12 is not more than the price at which customers would be willing to buy 120 cakes. Thus customers are willing to buy a total of 120 (and more) cakes at \$12. Each solution (q, p) represents a quantity of cakes q that customers would be willing to buy at the unit price p.

(c) In order to be a solution to both of the given inequalities, a point (q, p) must lie on or below the line $p = 20 - q/20$ and on or above the line $p = 11 + q/40$. Thus, the solution set of the given system of inequalities is the region shaded in Figure 1.11.

A point (q, p) in this region represents a quantity q of cakes that customers would be willing to buy, and that the bakery-owner would be willing to make and sell, at the price p.

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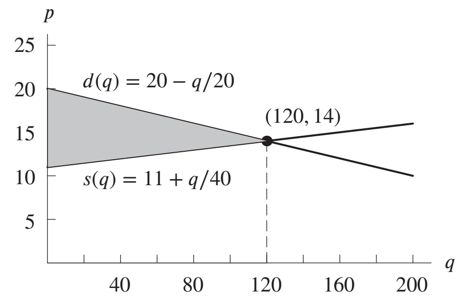

Figure 1.11: Possible cake sales at different prices and quantities

(d) To find the rightmost point of this region, we need to find the intersection point of the lines $p = 20 - q/20$ and $p = 11 + q/40$. At this point, p is equal to both $20 - q/20$ and $11 + q/40$, so these two expressions are equal to each other:

$$\begin{aligned} 20 - \frac{q}{20} &= 11 + \frac{q}{40} \\ 9 &= \frac{q}{20} + \frac{q}{40} \\ 9 &= \frac{3q}{40} \\ q &= 120. \end{aligned}$$

Therefore, $q = 120$ is the maximum number of cakes that can be sold at a price at which customers are willing to buy them all, and the owner of the bakery is willing to make them all. The price at this point is $p = 20 - 120/20 = 14$ dollars. (In economics, this price is called the *equilibrium price*, since at this point there is no incentive for the owner of the bakery to raise or lower the price of the cakes.)

70. (a) The line given by $(0, 2)$ and $(1, 1)$ has slope $m = \frac{2-1}{-1} = -1$ and y-intercept 2, so its equation is

$$y = -x + 2.$$

The points of intersection of this line with the parabola $y = x^2$ are given by

$$\begin{aligned} x^2 &= -x + 2 \\ x^2 + x - 2 &= 0 \\ (x + 2)(x - 1) &= 0. \end{aligned}$$

The solution $x = 1$ corresponds to the point we are already given, so the other solution, $x = -2$, gives the x-coordinate of C. When we substitute back into either equation to get y, we get the coordinates for C, $(-2, 4)$.

(b) The line given by $(0, b)$ and $(1, 1)$ has slope $m = \frac{b-1}{-1} = 1 - b$, and y-intercept at $(0, b)$, so we can write the equation for the line as we did in part (a):

$$y = (1 - b)x + b.$$

We then solve for the points of intersection with $y = x^2$ the same way:

$$\begin{aligned} x^2 &= (1 - b)x + b \\ x^2 - (1 - b)x - b &= 0 \\ x^2 + (b - 1)x - b &= 0 \\ (x + b)(x - 1) &= 0 \end{aligned}$$

Again, we have the solution at the given point $(1, 1)$, and a new solution at $x = -b$, corresponding to the other point of intersection C. Substituting back into either equation, we can find the y-coordinate for C is b^2, and thus C is given by $(-b, b^2)$. This result agrees with the particular case of part (a) where $b = 2$.

71. Looking at the given data, it seems that Galileo's hypothesis was incorrect. The first table suggests that velocity is not a linear function of distance, since the increases in velocity for each foot of distance are themselves getting smaller. Moreover, the second table suggests that velocity is instead proportional to *time*, since for each second of time, the velocity increases by 32 ft/sec.

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Strengthen Your Understanding

72. The slope of a linear function of x is given by the function's rise (Δy) over its run (Δx) over any interval. So, for $y = b+mx$, we have:

$$\text{Slope} = m = \frac{\Delta y}{\Delta x}.$$

73. The line $x = 3$ is vertical, so it is not a function of x: it fails the vertical line test. The line $y = 3$ is horizontal, hence a linear function of x with slope 0.

74. The line $y - 3 = 0$ or, equivalently, $y = 3$ is horizontal, hence has slope 0 in the xy-plane.

75. The line $y = 0.5 - 3x$ has a negative slope and is therefore a decreasing function.

76. If y is directly proportional to x we have $y = kx$. Adding the constant 1 to give $y = 2x + 1$ means that y is not proportional to x.

77. One possible answer is $f(x) = 2x + 3$.

78. One possible answer is $q = \dfrac{8}{p^{1/3}}$.

79. False. A line can be put through any two points in the plane. However, if the line is vertical, it is not the graph of a function.

80. True. Suppose we start at $x = x_1$ and increase x by 1 unit to $x_1 + 1$. If $y = b + mx$, the corresponding values of y are $b + mx_1$ and $b + m(x_1 + 1)$. Thus y increases by

$$\Delta y = b + m(x_1 + 1) - (b + mx_1) = m.$$

81. True. Solving for y on the second equation, we see that the second linear function has the same equation as the first:

$$\begin{aligned} x &= -y + 1 \\ x - 1 &= -y \\ -x + 1 &= y. \end{aligned}$$

82. False. Solving for y on the second equation, we get $y = (-1/2)x + 1$ which is a linear function with a different slope and intercept from the first equation.

83. False. For example, let $y = x + 1$. Then the points $(1, 2)$ and $(2, 3)$ are on the line. However the ratios

$$\frac{2}{1} = 2 \qquad \text{and} \qquad \frac{3}{2} = 1.5$$

are different. The ratio y/x is constant for linear functions of the form $y = mx$, but not in general. (Other examples are possible.)

84. False. For example, if $y = 4x + 1$ (so $m = 4$) and $x = 1$, then $y = 5$. Increasing x by 2 units gives 3, so $y = 4(3) + 1 = 13$. Thus, y has increased by 8 units, not $4 + 2 = 6$. (Other examples are possible.)

85. (b) and (c). For $g(x) = \sqrt{x}$, the domain and range are all nonnegative numbers, and for $h(x) = x^3$, the domain and range are all real numbers.

Solutions for Section 1.2

Exercises

1. The graph shows a concave up function.

2. The graph shows a concave down function.

3. This graph is neither concave up or down.

4. The graph is concave up.

5. Initial quantity = 5; growth rate = 0.07 = 7%.

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6. Initial quantity = 7.7; growth rate = −0.08 = −8% (decay).

7. Initial quantity = 3.2; growth rate = 0.03 = 3% (continuous).

8. Initial quantity = 15; growth rate = −0.06 = −6% (continuous decay).

9. Since $e^{0.25t} = \left(e^{0.25}\right)^t \approx (1.2840)^t$, we have $P = 15(1.2840)^t$. This is exponential growth since 0.25 is positive. We can also see that this is growth because $1.2840 > 1$.

10. Since $e^{-0.5t} = (e^{-0.5})^t \approx (0.6065)^t$, we have $P = 2(0.6065)^t$. This is exponential decay since −0.5 is negative. We can also see that this is decay because $0.6065 < 1$.

11. $P = P_0(e^{0.2})^t = P_0(1.2214)^t$. Exponential growth because $0.2 > 0$ or $1.2214 > 1$.

12. $P = 7(e^{-\pi})^t = 7(0.0432)^t$. Exponential decay because $-\pi < 0$ or $0.0432 < 1$.

13. (a) Let $Q = Q_0a^t$. Then $Q_0a^5 = 75.94$ and $Q_0a^7 = 170.86$. So

$$\frac{Q_0a^7}{Q_0a^5} = \frac{170.86}{75.94} = 2.25 = a^2.$$

So $a = 1.5$.

(b) Since $a = 1.5$, the growth rate is $r = 0.5 = 50\%$.

14. (a) Let $Q = Q_0a^t$. Then $Q_0a^{0.02} = 25.02$ and $Q_0a^{0.05} = 25.06$. So

$$\frac{Q_0a^{0.05}}{Q_0a^{0.02}} = \frac{25.06}{25.02} = 1.001 = a^{0.03}.$$

So

$$a = (1.001)^{\frac{100}{3}} = 1.05.$$

(b) Since $a = 1.05$, the growth rate is $r = 0.05 = 5\%$.

15. (a) The function is linear with initial population of 1000 and slope of 50, so $P = 1000 + 50t$.

(b) This function is exponential with initial population of 1000 and growth rate of 5%, so $P = 1000(1.05)^t$.

16. (a) This is a linear function with slope −2 grams per day and intercept 30 grams. The function is $Q = 30 - 2t$, and the graph is shown in Figure 1.12.

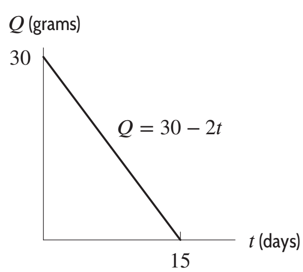

Figure 1.12

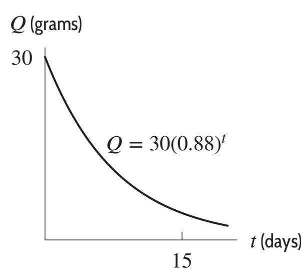

Figure 1.13

(b) Since the quantity is decreasing by a constant percent change, this is an exponential function with base $1-0.12 = 0.88$. The function is $Q = 30(0.88)^t$, and the graph is shown in Figure 1.13.

17. The function is increasing and concave up between D and E, and between H and I. It is increasing and concave down between A and B, and between E and F. It is decreasing and concave up between C and D, and between G and H. Finally, it is decreasing and concave down between B and C, and between F and G

18. (a) It was decreasing from March 2 to March 5 and increasing from March 5 to March 9.

(b) From March 5 to 8, the average temperature increased, but the rate of increase went down, from 12° between March 5 and 6 to 4° between March 6 and 7 to 2° between March 7 and 8.

From March 7 to 9, the average temperature increased, and the rate of increase went up, from 2° between March 7 and 8 to 9° between March 8 and 9.

Problems

19. (a) A linear function must change by exactly the same amount whenever x changes by some fixed quantity. While $h(x)$ decreases by 3 whenever x increases by 1, $f(x)$ and $g(x)$ fail this test, since both change by different amounts between

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$x = -2$ and $x = -1$ and between $x = -1$ and $x = 0$. So the only possible linear function is $h(x)$, so it will be given by a formula of the type: $h(x) = mx + b$. As noted, $m = -3$. Since the y-intercept of h is 31, the formula for $h(x)$ is $h(x) = 31 - 3x$.

(b) An exponential function must grow by exactly the same factor whenever x changes by some fixed quantity. Here, $g(x)$ increases by a factor of 1.5 whenever x increases by 1. Since the y-intercept of $g(x)$ is 36, $g(x)$ has the formula $g(x) = 36(1.5)^x$. The other two functions are not exponential; $h(x)$ is not because it is a linear function, and $f(x)$ is not because it both increases and decreases.

20. Table A and Table B could represent linear functions of x. Table A could represent the constant linear function $y = 2.2$ because all y values are the same. Table B could represent a linear function of x with slope equal to $11/4$. This is because x values that differ by 4 have corresponding y values that differ by 11, and x values that differ by 8 have corresponding y values that differ by 22. In Table C, y decreases and then increases as x increases, so the table cannot represent a linear function. Table D does not show a constant rate of change, so it cannot represent a linear function.

21. Table D is the only table that could represent an exponential function of x. This is because, in Table D, the ratio of y values is the same for all equally spaced x values. Thus, the y values in the table have a constant percent rate of decrease:

$$\frac{9}{18} = \frac{4.5}{9} = \frac{2.25}{4.5} = 0.5.$$

Table A represents a constant function of x, so it cannot represent an exponential function. In Table B, the ratio between y values corresponding to equally spaced x values is not the same. In Table C, y decreases and then increases as x increases. So neither Table B nor Table C can represent exponential functions.

22. Since f is linear, its slope is a constant

$$m = \frac{20 - 10}{2 - 0} = 5.$$

Thus f increases 5 units for unit increase in x, so

$$f(1) = 15, \quad f(3) = 25, \quad f(4) = 30.$$

Since g is exponential, its growth factor is constant. Writing $g(x) = ab^x$, we have $g(0) = a = 10$, so

$$g(x) = 10 \cdot b^x.$$

Since $g(2) = 10 \cdot b^2 = 20$, we have $b^2 = 2$ and since $b > 0$, we have

$$b = \sqrt{2}.$$

Thus g increases by a factor of $\sqrt{2}$ for unit increase in x, so

$$g(1) = 10\sqrt{2}, \quad g(3) = 10(\sqrt{2})^3 = 20\sqrt{2}, \quad g(4) = 10(\sqrt{2})^4 = 40.$$

Notice that the value of $g(x)$ doubles between $x = 0$ and $x = 2$ (from $g(0) = 10$ to $g(2) = 20$), so the doubling time of $g(x)$ is 2. Thus, $g(x)$ doubles again between $x = 2$ and $x = 4$, confirming that $g(4) = 40$.

23. We see that $\frac{1.09}{1.06} \approx 1.03$, and therefore $h(s) = c(1.03)^s$; c must be 1. Similarly $\frac{2.42}{2.20} = 1.1$, and so $f(s) = a(1.1)^s$; $a = 2$. Lastly, $\frac{3.65}{3.47} \approx 1.05$, so $g(s) = b(1.05)^s$; $b \approx 3$.

24. (a) This is the graph of a linear function, which increases at a constant rate, and thus corresponds to $k(t)$, which increases by 0.3 over each interval of 1.

(b) This graph is concave down, so it corresponds to a function whose increases are getting smaller, as is the case with $h(t)$, whose increases are 10, 9, 8, 7, and 6.

(c) This graph is concave up, so it corresponds to a function whose increases are getting bigger, as is the case with $g(t)$, whose increases are 1, 2, 3, 4, and 5.

25. (a) This is a linear function, corresponding to $g(x)$, whose rate of decrease is constant, 0.6.

(b) This graph is concave down, so it corresponds to a function whose rate of decrease is increasing, like $h(x)$. (The rates are $-0.2, -0.3, -0.4, -0.5, -0.6$.)

(c) This graph is concave up, so it corresponds to a function whose rate of decrease is decreasing, like $f(x)$. (The rates are $-10, -9, -8, -7, -6$.)

26. Graph (I) and (II) are of increasing functions, therefore the growth factor for both functions is greater than 1. Since graph (I) increases faster than graph (II), graph (I) corresponds to the larger growth factor. Therefore graph (I) matches $Q = 50(1.4)^t$ and graph (II) is $Q = 50(1.2)^t$.

Similarly, since graphs (III) and (IV) are both of decreasing functions, they have growth factors between 0 and 1. Since graph (IV) decreases faster than graph (III), graph (IV) has the smaller decay factor. Thus graph (III) is $Q = 50(0.8)^t$ and graph (IV) is $Q = 50(0.6)^t$.